Risk in Networked Systems

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Collaborators

- **Transportation**
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- **Power Grid**
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Cascades and Systemic Risk

Systemic Risk is a term used to describe fragility in interconnected systems that result in cascades of failures due to either relatively small shocks at the subsystem level or larger and more malicious types of disruptions affecting the whole system.

- Air Traffic Congestion: $31.2B
- Power Outages: $80B-$150B
- Financial Crisis 2008: $500B + ...
- Major Disruptions: Fukushima, H1N1
Resilience of Transportation Networks
Disturbances in Urban Transportation Networks

Typical Monday at 6:30 p.m.

Monday November 7, 2011, 6:30 p.m.

(Courtesy: Google Maps)
Objective:
Develop a dynamical model for transportation and derive metrics for their resilience
Outline

• Dynamical network flow formulation
• Stability of equilibria
• Margins of resilience
• Cascade effects
• Conclusions
Transportation as Network Flow

- Directed acyclic graph with single O/D pair
- Constant arrival rate $\lambda_{in}$ at the origin
- Driver route choice decisions + traffic physics determine $\lambda_{out}(t)$
Static Network Flow

- **Link flow capacity:** $f_i^{\text{max}}$

\[
\lambda_{\text{out}} = \lambda_{\text{in}} \Longleftrightarrow \text{feasible: } f \\
\lambda_{\text{out}} \leq f_i^{\text{max}} \quad \forall i \\
\sum_{\text{incoming}} f_i = \sum_{\text{outgoing}} f_j
\]

- **Max flow min cut theorem:**

\[
\lambda_{\text{in}} \leq \text{min-cut capacity} \implies \text{feasible } f
\]

- **Static perspective:** link outflow always equals inflow

\[
\text{cut } C
\]

\[
\lambda_{\text{in}} \quad i \\ v \\ \lambda_{\text{out}} = \lambda_{\text{in}} \\
\text{node cut}
\]
Wardrop Equilibrium

- $\pi$ : distribution of driver population by route preference
- $\pi$ induces static $f^\pi$

Wardrop equilibrium:

- $\text{delay}(f)$ on any used path is no greater than the delay on any other path
- globally stable under best response dynamics if $\lambda_{in} < \text{min-cut capacity}$
- $\pi$ (and hence $f^\pi$) evolves as per global best response strategy by drivers
Transportation physics

- Congestion dynamics

Rate of change of \( \rho_i \) = flow into link \( i \) – flow out of link \( i \)

- Flow conservation

\[
\sum_{i \text{ incoming to } v} f_i = \sum_{j \text{ outgoing from } v} f_j \quad \forall v
\]
Flow function

- Outflow on a link depends on the traffic density on that link: $f_j(\rho_j)$

\[ f_i(\rho_i) \]
\[ f_i^{\text{max}} \]
\[ \rho_i \]

Outflow is not necessarily equal to inflow on a link

$\rho_i$ : density on link
Multi-scale driver decision model

- Drivers take decision at every node
- Node-wise decisions influenced by:
  - global information available infrequently
  - real-time node-specific information
Local route choice decisions

- At node $v$, $G: \{\rho, \pi\} \rightarrow \text{prob. vector}$
  - local info
  - (old) global info

- Locally responsive routing policy $G^*$:
  - Consistency: $G^*_i(\rho^{\pi}, \pi) \sim \pi$
    - if local observations match expectation, then follow suit
  - Sensitivity:
    - locally prefer links with less congestion
    $$\frac{\partial G^*_i}{\partial \rho_j} \geq 0, \quad i \neq j$$
Dynamical network flow

- Congestion dynamics (fast scale)
  \[
  \dot{\rho}_i(t) = \text{inflow at } v \cdot G_i(\rho, \pi) - f_i(\rho_i)
  \]

- Global decision dynamics (slow scale)
  \[
  \dot{\pi} = \eta \left( \text{best response}(\rho) - \pi \right)
  \]

- Flow conservation
  \[
  \sum_{i \text{ incoming to } v} f_i = \sum_{j \text{ outgoing from } v} f_j \quad \forall v
  \]
Stability of Wardrop equilibrium

Theorem: If

- \( \lambda_{\text{in}} < \) min-cut capacity
- Drivers do not update their global decisions sufficiently fast w.r.t. traffic dynamics (small \( \eta \))

- Then Wardrop equilibrium is globally stable.
Perturbations: infinite density capacity

\[ \delta_i = \| f_i - \tilde{f}_i \|_{\infty} \]

\[ f_i(\rho_i) : t = 0 \]

\[ f_i(\rho_i) : t > 0 \]

\[ \delta = \sum_{i \in \mathcal{E}} \delta_i \]
Transferring Property

- The perturbed network is fully transferring w.r.t. $\text{eqm} f^{eq}$ (not necessarily Wardrop) under $G$ if:
  \[
  \lim \inf_{t \to \infty} \lambda_{\text{out}}(t) = \lambda_{\text{in}} \quad \text{with initial condition } f^{eq}
  \]

- Margin of resilience for a given $G$ and $f^{eq}$
  
  := $\inf_{\delta}$ perturbed network is not fully transferring w.r.t. $f^{eq}$ under $G$
Upper Bound on Margin of Resilience

\[ \forall G, \text{ margin of resilience } \leq \min \text{ cut residual capacity} \]

\[ := \min_{\text{cut } C} \sum_{i \in C} (f_{i}^{\text{max}} - f_{i}^{\text{eq}}) \]
A Tighter Upper Bound

• $\forall G$, margin of resilience $\leq \min \text{ node cut residual capacity}$

$$\min_v \sum_{i \text{ outgoing from } v} (f_i^{\max} - f_i^{eq})$$
Sufficiency for Margin of Resilience

Possible loss of resilience due to:

- Passive routing

- Aggressive routing
Optimality of Locally Responsive Routing

• $G^*$ creates the perfect balance between passive and aggressive routing

• For $G^*$, margin of resilience = $\min_v$ residual capacity of node $v$
Perturbations: finite density capacity

\[ \dot{\rho}_i = \mathbf{1}_{\text{link } i \text{ open}} \cdot \text{inflow at node } v \cdot G_i - \mathbf{1}_{\text{downstream open}} \cdot f_i \]

Finite density capacity constraints cause upstream cascades
Upstream Cascades
Upstream Cascades can Increase Resilience

Unbounded density capacity

Upstream cascades due to bounded density capacity

Upstream cascades compensate for lack of downstream information
Implications for Intelligent Transportation Systems

- Green light control
  - to influence routing $G$

- Congestion pricing
  - to influence equilibrium

- Automated driving
  - to influence the flow function
Conclusions

• Dynamical model for transportation networks

• Stability of equilibria under multiscale driver decisions

• Robust route choice behavior

• Characterization of margins of resilience

• Effect of cascades on the margins
Future Power Grid
Economics of Outages

- Power outages cost US economy $80B - 150B annually (0.01 % of GDP)
The Smart Grid

Generators

Utilities

Consumers

Transmission

Distribution

ISO

Market

Real Time Demand Response

Real Time Demand Pricing and Response

Real Time Pricing
Price Volatility: It does get even more dramatic...

There are many examples like this.
Dynamic Retail Pricing of Electricity

1. Time of use Pricing  
2. Critical Peak Pricing  
3. Real Time Pricing

Borenstein et al, 2002¹:
“We conclude by advocating much wider use of dynamic retail pricing, under which prices faced by end-use customers can be adjusted frequently and on short notice to reflect changes in wholesale prices.”

“…Such price-responsive demand holds the key to mitigating price volatility in wholesale electricity spot markets.”

W. Hogan, 2010²:
“…any consumer who is paying the RTP for energy is charged the full LMP for its consumption and avoids paying the full LMP when reducing consumption.”

“Expanding the use of dynamic pricing, particularly real-time pricing, to provide smarter prices for the smart grid would be a related priority….”
Real-Time Demand Response
Closing the Loop

ISO

Dispatch

Demand prediction

$dt$

Consumer Model

Supply Schedule

Locational marginal price

wind
Stability vs. Volatility

price vs. time

- Deferred consumption

price vs. time

- Synchronized consumption
RTP Increases Volatility
Value of Anarchy

- **Price of Anarchy**: Loss in efficiency due to strategic interactions in contrast to a coordination

- Simple model: one agent with shiftable demand and another with instantaneous demand

- Contrast optimal efficient solution to a Stackelberg game of strategic behavior

- A new tradeoff: Cooperation can increase endogenous risk
Setup

![Diagram showing inflexible and flexible load over time]

Inflexible load

Flexible load

- $t$
- $t+1$
- $t+2$
- $t+3$
Model

System state:

Aggregate unshiftable loads \( x(t) \)

\[
x(t) = \begin{cases} 
\text{aggregate unshiftable} & d_1(t) \\
\text{unshiftable arrival at current period} & d_2(t - 1) - u(t - 1)
\end{cases}
\]

Consumer arrival with shiftable load \( d_2(t) \)

Load shifting decision:

Only 1 decision maker at \( t \) : the new arrival with shiftable load

Split load into two periods \( (t, t + 1) \) based on \( (x(t), d_2(t)) \)

\( (u(t), d_2(t) - u(t)) \)
Problem Formulation

• Deadline constraints on demands:
  \[ \sum_{t \text{ in } i^{\text{th}} \text{ active window}} u_{t,i} = i^{\text{th}} \text{ work load} \]

• Endogenous prices couple individual decisions:
  \[ p_t \propto \sum_i u_{t,i} \]

• **Non-cooperative** decision making:
  \[ \min_{u_{t,i}} p_t u_{t,i} + \mathbb{E}[p_{t+1} u_{t+1,i}] \]

• **Cooperative** decision making:
  \[ \min_{[u_{t,i}]} \mathbb{E} \left[ \text{time average of } \sum_i p_t u_{t,i} \right] \]
Symmetric Markov Perfect equilibrium in dynamic stochastic game

\[ u^s(x(t), d_2(t)) = \arg \min_u \{ p(t)u + \mathbb{E}_t[p(t+1)(d_2(t) - u)] \} \]

\[ p(t) = x(t) + u \]
\[ p(t+1) = x(t+1) + u^s(x(t+1), d_2(t+1)) \]

Overlapping type 2 consumers

Flavor of Stackelberg competition
Solution: Strategic

Symmetric Markov Perfect equilibrium in dynamic stochastic game

\[ u^s(x(t), d_2(t)) = \arg \min_u \{ p(t)u + E_t[p(t+1)(d_2(t) - u)] \} \]

**Equilibrium strategy**

Unique MPE with linear stationary equilibrium strategy:

\[ u^s(x, d_2) = -\frac{1}{2(1 + \sqrt{1 - \frac{q^2}{2}})} x + \frac{1}{1 + \frac{1}{\sqrt{1 - \frac{q^2}{2}}}} d_2 + \frac{q_1 \mu_1 + q_2 \mu_2 \frac{1}{1 + \sqrt{1 - \frac{q^2}{2}}}}{2(1 + \sqrt{1 - \frac{q^2}{2}})} \]
Solution: Cooperative

Bellman equation for infinite horizon average cost MDP

\[ \lambda^c + V^c(x) = (1 - q_2)(x^2 + \mathbb{E}[V^c(d_1)]) + q_2\mathbb{E}[\min_u \{(x + u)^2 + V^c(d_2 - u + d_1)\}] \]

Optimal stationary policy

There exists an optimal linear stationary policy:

\[ u^c(x, d_2) = -\frac{1}{1 + \sqrt{1 - q_2}^{a^c}} x + \frac{1}{1 + \sqrt{1 - q_2}^{b^c}} d_2 + \frac{q_1 \mu_1 + q_2 \mu_2}{1 + \sqrt{1 - q_2}^{e^c}} \]
Welfare impacts

Under linear stationary policy \( u(x, d_2) = -ax + bd_2 + e \)

Efficiency

Variance
\[
- \frac{1}{2} E[U(t)^2] = - \frac{1}{2} \lambda
\]

Risk

Tail probability \( \Pr(x(t) \geq M) \)

\( \mathcal{X} = \mathcal{X}_k \) with probability \( q_2^k (1 - q) \)

\[
E[\mathcal{X}_k] = \frac{(1 - a^{k+1} \mu_1 + (1 - a^k)((1 - b)\mu_2 - e))}{1 - a}
\]

\[
Var[\mathcal{X}_k] = \frac{(1 - a^{2(k+1)}) \sigma_1^2 + (1 - a^{2k})(1 - b)^2 \sigma_2^2}{1 - a^2}
\]

Strategic
\( u^s(x, d_2) = -a^s x + b^s d_2 + e^s \)

Cooperative
\( u^c(x, d_2) = -a^c x + b^c d_2 + e^c \)
Price of Anarchy: what about risk?

Aggregate demand sample path

![Graph showing aggregate demand for small and large time scales with cooperative and non-cooperative scenarios. The graph highlights spikes in demand.]

Cooperative
Non-cooperative
Example I: $L = 2$

Aggregate demand stationary distribution

- Low variance
- Spikes

**Empirical PDF**

- Y-axis Linear scale
- Y-axis Log scale

**Graphs**

- Cooperative
- Non-cooperative
Message

- **Feedback**: May have unintended consequences

- **Value of Anarchy**: Strategic behavior results in less risk (more robustness) than an optimized behavior by a planner

- Classical performance-robustness tradeoff

- Tradeoff should be achieved by a different market mechanism

- Market architecture is critical
Innovation

- Robust Architecture (Distributed vs decentralized)
  - Robust Feedback
  - Distributed computing, Strategic decisions
  - Network effects and Resilience

- Degree of Regulation and Centralization

- Incentives
  - Who owns what?
  - Can renewables be penalized?
  - Storage

- Electrification of cars
Conclusions

• Research in Optimality vs Risk
  • Next step from robust control theory

• Two characteristics
  • Many decision makers
  • Network effects

• Value of Anarchy
  • Classical tradeoff between optimality and robustness

• Resilience of Physical networks
  • Conditions on both classes of decisions and networks
Other Work

• Systemic Risk in Financial Systems

• Cascaded behavior in social phenomenon
  • Information structure
  • Global games
Boom-Bust

Payoff realized

Short-term creditor

Noisy signal about payoff

Pay back creditors or default

Pay departing creditors

BANK

Liquidate positions at fire-sale discount

Long-term creditor

Pay back creditors or default

Departing creditors

Pay departing creditors

Noisy signal about payoff

Payoff realized

Investors

Boom-Bust
Connection

To understand these disparate societal problems, we need to understand the interactions among complex systems.
Thank You