Unbiased Estimation of Causal Effects under Network Interference
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Questions
- How can one account for interference between units in an online social network experiment?
- What are the collection of unbiased estimators for causal effects and how can one choose among them?
- How do different aspects of the potential outcomes and the network impact performance?

Potential Outcomes Framework (Neyman-Rubin Causal Model)
- An allocation is a vector of \( \{0, 1\} \)-treatments assignments, \( \mathbf{z} = (z_1, z_2, \ldots, z_n) \in \{0, 1\}^n \).
- Design is a probability distribution \( p \) on \( \{0, 1\}^n \), with \( \mathbf{z}^{obs} \sim p \).
- For allocation \( \mathbf{z} \), the outcome for unit \( i \) is \( Y_i(\mathbf{z}) \).
- Causal estimand is some function of the potential outcomes.
  - e.g. the average effect of total treatment is \( \frac{1}{n} \sum_{i=1}^{n} Y_i(1) - Y_i(0) \).
  - Core challenge of causal inference is only \( Y_i(\mathbf{z}^{obs}), \ldots, Y_n(\mathbf{z}^{obs}) \) are observed.

What is network Interference?
No Interference (SUTVA) \( Y_i \) depends only on \( z_i \).
Neighborhood Interference Assumption \( Y_i \) depends only on \( z_i \) and \( z_j \) for \( j \) in the neighborhood of \( i \).
- Neighborhood is given by a (directed) network \( g \in \{0, 1\}^{n \times n} \).

Additional Assumption
- Additivity of Main Effects Effect of own treatment and neighbors' treatments are additive.
- Additivity of Interference Effects Effects of each neighbor are additive.
- Symmetrically Received Interference Outcome only depends on number of treated neighbors.
- Symmetrically Sent Interference Effect of treatment of unit \( j \) is the same for each unit with \( j \) as a neighbor.

An example parameterization: SANIA
If we assume additivity of main effects and symmetrically received interference effects:
\[
Y_i(\mathbf{z}) = \alpha_i + \beta_i z_i + \Gamma_i(d_i^z)
\]
where
- \( \alpha_i \in \mathbb{R} \) is the baseline outcome,
- \( \beta_i \in \mathbb{R} \) is the “direct treatment effect”,
- \( \Gamma_i : [n-1] \rightarrow \mathbb{R} \) is the interference effect function, and
- \( d_i^z \) is the number of treated neighbors of \( i \).
Suppose we want to estimate \( \bar{\beta} = \frac{1}{n} \sum_i \beta_i \).

When do unbiased estimate of \( \bar{\beta} \) exist?
We consider linear estimates of the form
\[
\hat{\beta}_w = \sum_i w_i(z^{obs}) Y_i(z^{obs}).
\]

Proposition: LUEs exist under SANIA if and only if for each \( i \in [n] \) there exist allocations \( \mathbf{z}, \mathbf{z}' \) such that \( p(\mathbf{z}) > 0, p(\mathbf{z}') > 0, d_i^\mathbf{z} = d_i^{\mathbf{z}'} \), \( z_i = 1 \), and \( z_i' = 0 \).

Minimum Integrated Variance LUEs
- We propose minimum integrated variance (MIV) LUE for some “prior” distribution over the parameters \( \alpha_i, \beta_i, \Gamma_i \).
- For priors with independence across units, the MIV LUE is Horvitz-Thompson like, with inverse propensity score weighting.
- For priors where parameters are equal across units, the MIV LUE is similar to a naive estimate, where one takes the difference of means between the treatment and control groups, stratified on the number of treated neighbors.

Figure: Mean square errors on the log scale versus the number of units, for dense graphs and sparse graphs. Independent is a MIV LUE where all parameters are independent. Equal is a MIV LUE where parameters are equal across units. Naive and Horvitz-Thompson are the two standard estimators under no interference and Stratified Naive an average of naive estimators conditioned on the number of treated neighbors.

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